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On the Quantum Field Theoretic Origin of a Class of Brans–Dicke Scalars

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Within the context of a manifestly covariant massive Abelian gauge field model, it is demonstrated that a mass is genuinely compatible with the existence of a bonafide Goldstone boson. Some arguments have been placed to claim that this Goldstone boson may occupy a physical sector of the Hilbert space and as a consequence, at the zero momentum limit, is shown to connect the spectral measure to the bare coupling constant through the Higgs meson mass. It is conjectured that this Goldstone boson can be recognized to invoke a class of Brans–Dicke scalars of the scalar–tensor theory of gravitation.

1. INTRODUCTION

We shall consider a model, a variant of the Stückelberg formalism for massive Abelian gauge fields interacting with a conserved current (Ghose and Das, 1972). In this model, the redundant spin-zero component for the vector field $V_\mu(x)$ has been made to mix with a free, negative metric, spinless Lagrange multiplier field $B(x)$, and is decoupled from the theory by virtue of the equation of motion. The unitarity of the S matrix is defined through $B^{(+)}|\text{phys}\rangle = 0$. It is observed that the squared mass m_0^2 of the vector meson acquires a mass M^2 from the spontaneous breaking of a one-parameter global symmetry and also a genuine Goldstone boson is present in the theory. This Goldstone boson is shown to satisfy a free field equation and at the same time it has been stressed that it can be used to set up relations between the n- and $(n+1)$-point vacuum amplitudes where the extra particle is always in the limit of zero momentum. Thus a connection between the spectral measure and the coupling constant is obtained. We propose this Goldstone boson as a member of a class of Brans–Dicke scalars of the scalar–tensor theory of gravitation. The present model is a manifestly covariant one and the spin-zero component of the vector field always possesses a mixing with the negative metric field $B(x)$ and is decoupled by virtue of the equation of motion. This guarantees that scale transformation and Lorentz covariance are always maintaining compatibility in this model (Das and Ghose, 1973).

2. SPONTANEOUS SYMMETRY BREAKING

Let us define an automorphism α of the field algebra F, commuting with the proper Poincaré group, by the set of local transformations $(i=1,\ldots,N)$

$$A_i \to A_i^{\alpha} \tag{2.1}$$

and suppose that this transformation corresponds to an n-parametric group G of fields transforming in this way:

$$A_i(x) \to A_i^{g(\lambda)}(x) \tag{2.2}$$

where $g(\lambda)=g(\lambda_1,\ldots,\lambda_n)\in G$. If the generator of (2.2) is some local conserved current j_μ, then its relation with $A_i(x)$ may be represented in this way:

$$\frac{d}{d\lambda_a} A_i^{g(\lambda)}(x') = i\left[\int d^3x\, f_R(\mathbf{x}) j_a^0(\mathbf{x},t),\, A_i(x')\right] \tag{2.3}$$

where $f_R(\mathbf{x})$ is test function which is 1 for $|\mathbf{x}|<R+\epsilon$ and vanishes for $|\mathbf{x}|>R+\epsilon$. If the theory involves spontaneous symmetry breaking,

$$\lim_{R\to\infty} \langle \Omega | [Q_{R,t}^a, A] | \Omega \rangle \neq 0 \tag{2.4}$$

where $Q_R^a(t)=\int d^3x\, f_R(\mathbf{x}) j_a^0(\mathbf{x},t)$, A is any field algebra F, and Ω is the true vacuum. The essential content of the Goldstone theorem (Goldstone *et al.*, 1962) is that the condition given by (2.4) occurs (Reeh, 1968) when there is a singularity in the spectral representation of $\langle \Omega | [j^0(x), A] | \Omega \rangle$, i.e., from zero-mass one-particle intermediate states (Ezawa and Swieca, 1967).

If we now write E_1 for the projector on the zero-mass one-particle states and

$$F_a^{\mu}(x) \equiv \langle \Omega | j_a^{\mu}(x) E_1 A - A E_1 j_a^{\mu}(x) | \Omega \rangle \tag{2.5}$$

then for (2.3) we can write

$$i\left\langle \Omega\left|\frac{dA}{dt}\right|\Omega\right\rangle = \lim_{R\to\infty} \langle \Omega|[Q^a_{R,t}, A]|\Omega\rangle$$

$$= \lim_{R\to\infty} \langle \Omega|Q^a_{R,t} E_1 A - A E_1 Q^a_{R,t}|\Omega\rangle$$

$$= \lim_{R\to\infty} \int d^3x\, f_R(\mathbf{x}) F^0_a(x) \qquad (2.6)$$

We note that actually,

$$F^\mu_a(x) = \int d^3x' \{\Delta(\mathbf{x}-\mathbf{x}', t)\rho^\mu_1(\mathbf{x}') + \dot{\Delta}(\mathbf{x}-\mathbf{x}', t)\rho^\mu_2(\mathbf{x}')\} \qquad (2.7)$$

which is the Jost–Kallén–Lehman–Dyson representation for (2.5). The physical meaning of (2.6) may be summerized (Picasso and Ferrari, 1970) by the formula

$$\left\langle \Omega\left|\frac{dA}{dt}\right|\Omega\right\rangle = (2\pi)^{3/2} F^0_a \langle a|A|\Omega\rangle \qquad (2.8)$$

which can be used to set up connections between the n- and $(n+1)$-point vacuum amplitudes, i.e., between vertices and scattering amplitudes, spectral functions and vertices, and so on, of course, the extra particle *being always in the limit of zero momentum*.

3. THE MODEL

We are going to explore a manifestly covariant, renormalizable, massive Abelian gauge model using a variant of the Stückelberg formalism. In this formalism the redundant spin-zero component of the vector field $V_\mu(x)$ has been made to mix with a free, negative metric, spinless field and is decoupled from other sources by virtue of the equation of motion. The spin-zero field $B(x)$ is actually a Lagrange multiplier field operator. The unitarity of the S matrix is guaranteed, because it is defined in the physical sector alone through $B^+|\text{phys}\rangle = 0$.

3.1 Quantization. Let us consider a Lagrangian describing the interaction of a massive vector field $U_\mu(x)$, with a bare mass m_0, with a conserved current of a spin-zero charged matter field $\phi'(x)$ having a double-hump self-interaction:

$$\mathcal{L}' = -\tfrac{1}{4}U_{\mu\nu}U^{\mu\nu} + \tfrac{1}{2}m_0^2 U_\mu U^\mu - \tfrac{1}{2}\partial_\mu B\partial^\mu B$$
$$+(\partial_\mu + ig_0 U_\mu)\phi'^*(\partial^\mu - ig_0 U^\mu)\phi' + \mu_0^2|\phi'|^2 - \lambda|\phi'|^4 \tag{3.1}$$

Let us now perform the Stückelberg decomposition

$$U_\mu(x) = V_\mu(x) + \frac{1}{m_0}\partial_\mu B(x) \tag{3.2a}$$

and the gauge transformation

$$\phi'(x) = \exp\{i(g_0/m_0)B(x)\}\phi(x) \tag{3.2b}$$

and get the equation of motion for the unrenormalized field $V_\mu(x)$:

$$\partial^\nu V_{\nu\mu} + m_0^2 V_\mu = -j_\mu - m_0\partial_\mu B \tag{3.3}$$

and

$$\partial_\mu V^\mu = 0 \tag{3.4}$$

The Noether current j_μ is given by

$$j_\mu = \frac{\delta\mathcal{L}'}{\delta\phi_{,\mu}}(-ie\phi) + \frac{\delta\mathcal{L}'}{\delta\phi'_{,\mu}}(ie\phi)$$
$$= ig_0\left[\phi^*(\partial_\mu + ig_0 V_\mu)\phi - \phi(\partial_\mu - ig_0 V_\mu)\phi^*\right] \tag{3.5}$$

also,

$$\partial_\mu j^\mu = 0 \tag{3.6}$$

From equations (3.3), (3.4), and (3.6) we recover the equation

$$\Box B = 0 \tag{3.7}$$

We assert that the scalar field $B(x)$ always satisfies the homogenous equation (3.7), so that

$$B(x) = -\int d^3x'\, D(x-x')\overleftrightarrow{\partial}_{x'} B(x') \tag{3.8}$$

A few nontrivial equal-time commutations are

$$\left[\dot{V}_\mu(x), V_\nu(x')\right]_{ET} = i(g_{\mu\nu} - g_{\mu 0} g_{\nu 0})\delta^3(\mathbf{x}-\mathbf{x}') \tag{3.9}$$

$$\left[V_\mu(x), \phi(x')\right]_{ET} = 0 \tag{3.10}$$

$$\left[V_\mu(x), B(x')\right]_{ET} = (-i/m_0) g_{\mu 0}\delta^3(\mathbf{x}-\mathbf{x}') \tag{3.11}$$

$$\left[V_\mu(x), \mathring{\phi}(x')\right] = 0 \tag{3.12}$$

$$\left[V_\mu(x), \mathring{B}(x')\right]_{ET} = (i/m_0) g_{\mu 0}\delta^3(\mathbf{x}-\mathbf{x}') \tag{3.13}$$

Consequently,

$$\left[B(x), j_\mu(x')\right] = 0 \tag{3.14}$$

independent of any model of the charged matter field. The momentum space propagators are

$$\overline{V_\mu^{(in)} V_\nu^{(in)}} = \frac{-i\left(g_{\mu\nu} - p_\mu p_\nu / p^2\right)}{p^2 - m_0^2 + i\epsilon} \tag{3.15a}$$

$$\overline{B^{(in)} B^{(in)}} = \frac{-i}{p^2 + i\epsilon} \tag{3.15b}$$

$$\overline{V_\mu^{(in)} B^{(in)}} = \frac{i p_\mu}{m_0^2 (p^2 + i\epsilon)} \tag{3.15c}$$

The relation (3.15a) shows that our Lagrangian (3.1) supported by (3.2) defines the same on-the-mass-shell S matrix as a similar Lagrangian without the B field. Moreover, the unitarity is preserved because of (3.7). In fact, we define states by

$$B^{(+)}|\text{phys}\rangle = 0 \tag{3.16}$$

along with $(d/dt)B^{(+)}|\text{phys}\rangle = 0$ at $t = 0$. Then (3.7) guarantees that if there are no B particles initially, there will be none finally. They do not take part in scattering. The S matrix defined in the physical sector of the Hilbert space is perforce unitary.

3.2 Spontaneous Symmetry Breaking. Let us now form real fields ϕ_1 and χ using ϕ and ϕ^* present in (3.1). Thus

$$\phi_1 = \frac{1}{2^{1/2}}(\phi + \phi^*), \qquad \chi = \frac{-i}{2^{1/2}}(\phi - \phi^*) \tag{3.17}$$

so that

$$\langle \Omega | \chi | \Omega \rangle = 0 \tag{3.18}$$

but

$$\langle \Omega | \phi_1 | \Omega \rangle = v \neq 0 \tag{3.19}$$

Let $\psi_1 = \phi_1 - v$. Then the total Lagrangian in terms of ψ and χ is

$$L'_{(\text{total})} = L'_{(\text{free})} + L'_{(\text{interaction})}$$

where

$$\begin{aligned} L'_{(\text{free})} = & -\tfrac{1}{4} U_{\mu\nu} U^{\mu\nu} + \tfrac{1}{2} m_0^2 U_\mu U^\mu - \tfrac{1}{2} \partial_\mu B \partial^\mu B + \tfrac{1}{2} (g_0 v)^2 U_\mu U^\mu \\ & + \tfrac{1}{2} \partial_\mu \chi \partial^\mu \chi + \tfrac{1}{2} \partial_\mu \psi \partial^\mu \mu \psi + (g_0 v) U_\mu \partial^\mu \chi \\ & - \tfrac{1}{2} (3\lambda^2 v^2 - \mu_0^2) \psi - \tfrac{1}{2} (\lambda^2 v^2 - \mu_0^2) \chi^2 \end{aligned} \tag{3.20a}$$

and

$$\begin{aligned} L'_{(\text{interaction})} = & \, g_0 (g_0 v) U_\mu U^\mu \psi + g_0 U_\mu (\psi \partial^\mu \psi) + \tfrac{1}{2} g_0^2 U_\mu U^\mu (\psi^2 + \chi^2) \\ & - (\lambda/4)(\psi^2 + \chi^2) - \lambda^2 g_0 \psi (\psi^2 + \chi^2) + (\mu_0^2 v - \lambda^2 v^3) \end{aligned} \tag{3.20b}$$

The condition $\langle \Omega | \psi | \Omega \rangle$ imposes $\mu_0^2 = \lambda^2 v^2$ at the zeroth order and we can put $M = gv$. Inserting M in $L'_{(\text{total})}$ we now find that U_μ has a mass $(m_0^2 + M^2)^{1/2}$ [see Equation (3.21a)]. Let us perform the Stückelberg decomposition

$$U_\mu = V_\mu + \frac{1}{(m_0^2 + M^2)^{1/2}} \partial_\mu B$$

Then for $L'_{(\text{free})}$ and $L'_{(\text{interaction})}$ we can write, respectively,

$$\begin{aligned} L_{(\text{free})} = & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(m_0^2 + M^2)V_\mu V^\mu \\ & + (m_0^2 + M^2)^{1/2} V_\mu \partial^\mu B + \tfrac{1}{2}\partial_\mu \chi \partial^\mu \chi \\ & + M\partial_\mu \chi V^\mu + \tfrac{1}{2}\partial_\mu \psi \partial^\mu \psi \\ & + MA^\mu \partial_\mu \psi - \mu_0^2 \psi^2 \end{aligned} \tag{3.21a}$$

and

$$\begin{aligned} L_{(\text{interaction})} = & \, g_0 M V_\mu V^\mu \psi + g_0 V_\mu (\psi \overleftrightarrow{\partial}^\mu \chi) \\ & + \tfrac{1}{2} g_0^2 V_\mu V^\mu (\psi^2 + \chi^2) - (\lambda^2/4)(\psi^2 + \chi^2)^2 - \lambda^2 g_0 \psi (\psi^2 + \chi^2) \end{aligned} \tag{3.21b}$$

The equation of motion in the lowest order theory are

$$\partial^\nu V_{\nu\mu} + m_1^2 V_\mu = -g_0 J_\mu - m_1 \partial_\mu B - M\partial_\mu \chi \tag{3.22a}$$

and

$$\partial_\mu V^\mu = 0 \tag{3.22b}$$

with

$$m_1^2 = m_0^2 + M^2$$

and

$$J_\mu = 2g_0 M V_\mu \psi + g_0 (\psi \overleftrightarrow{\partial}_\mu \chi) + g_0^2 V_\mu (\psi^2 + \chi^2) \tag{3.23}$$

and

$$\partial_\mu J^\mu = 0 \tag{3.24}$$

Also, in the zeroth order,

$$\Box \chi = 0 \tag{3.25}$$

and

$$(\Box+\mu^2)\psi=0 \tag{3.26}$$

where $\mu^2=2\mu_0^2$. For (3.22a) we can write

$$\partial^\nu V_{\nu\mu}+m_1^2 V_\mu=-g_0 J_\mu-m_1\partial_\mu\left[B+\frac{M}{m_1}\chi\right]=-g_0 J_\mu-m_1\partial_\mu B_1 \tag{3.27}$$

where $B_1=B+(M/m_1)\chi$ and in view of (3.7) and (3.25) we introduce

$$\Box B_1=0 \tag{3.28}$$

In the algebric condition $B_1=B+(M/m_1)\chi$, B occupies some unphysical sector of the Hilbert space, while χ is implemented in the physical sector of the Hilbert space [see the discussion following equation (4.1)]. Thus it is not desirable to introduce a transformation $B\to B-(M/m_1)\chi$ so that χ is shown to be transformed away from the theory. The following relations are now interesting:

$$[B_1(x),\psi(x')]=-(i/m_1)g_0\chi(x)D(x-x') \tag{3.29}$$

and

$$[B_1(x),\chi(x')]=-(iM/m_1)D(x-x') \tag{3.30}$$

We also note that

$$[J_\mu(x),B_1(x')]=\left(\frac{iM}{m_1}\right)g_0\{-\psi\partial_\mu D(x-x')+\partial_\mu\psi D(x-x')\} \tag{3.31}$$

We intend to calculate

$$\langle\Omega|[J_\mu(x),\chi(0)]|\Omega\rangle \tag{3.32}$$

The most general form of the Fourier transform of (3.32) can be expressed in this form:

$$\langle\Omega|[J\mu(x),\chi(0)]|\Omega\rangle=k^\mu\delta(k^2)\lambda(nk)[k^2(n)-k(nk)]\rho(k^2,nk)$$
$$+\eta^\mu\delta(\eta k)\Delta(k^2)+c\eta^\mu\delta^4(k) \tag{3.33}$$

Here c is constant and λ, ρ, and Δ are arbitrary functions of their indicated arguments. However, explicit model dependence due to physical requirements assures that the fourth term should not remain and bonafide Goldstone bosons come from the first two terms only. In this connection it is important to note that we should disregard the contribution from the third term. It corresponds to the intermediate states with dispersion of the form $k^0=0$ for all k. Its existence will guarantee that it is possible to generate from any given vacuum, other vacua, which have zero energy but nonzero momenta. This may imply spontaneous breakdown of translational invariance. Now in view of (3.25) we can write (Hagedorn, 1964)

$$\begin{aligned} D^{(+)}(x) &= \langle\Omega|\chi(x)\chi(0)|\Omega\rangle \\ &= -a_1 \ln\left\{-(x_0-i\epsilon)^2+\mathbf{x}^2\right\} \\ &\quad + a_2\left\{x^2-i\epsilon(x_0)\right\}^{-1}+a_3 \end{aligned} \tag{3.34}$$

where

$$\begin{aligned} \ln\left\{-(x_0-i\epsilon)^2+\mathbf{x}^2\right\} &= \ln\left|-(x_0-i\epsilon)^2+\mathbf{x}^2\right| \\ &\quad + i\arg\left\{-(x_0-i\epsilon)^2+\mathbf{x}^2\right\} \end{aligned} \tag{3.35}$$

with

$$-\pi<\arg\left\{-(x_0-i\epsilon)^2+\mathbf{x}^2\right\}\leqslant\pi$$

so that

$$[\chi(x),\chi(0)]=2\pi i\epsilon(x_0)\left\{a_1\theta(x^2)+a_2\delta(x^2)\right\} \tag{3.36}$$

where a_1 and a_2 are two model-dependent contants. Some usable relations in this connection are

$$\Box\ln\left\{-(x_0-i\epsilon_0)^2+\mathbf{x}\right\}=4\left(x^2-i\epsilon x_0\right)^{-1} \tag{3.37}$$

$$\Box\epsilon(x_0)\theta(x^2)=4\epsilon(x_0)\delta(x^2) \tag{3.38}$$

and

$$\partial_\mu\partial_\nu\partial_\rho\epsilon(x_0)\theta(x^2)|_{x_0=0}=8\pi g_{\mu0}g_{\nu0}g_{\rho0}\delta^3(\mathbf{x}) \tag{3.39}$$

Then in our model very simply it follows that

$$\left\langle\Omega\left|\left[J_\mu(x),\chi(0)\right]\right|\Omega\right\rangle=(\text{numerical constant})\partial_\mu\epsilon(x_0)\delta(x^2) \tag{3.40}$$

The Fourier transform of the right-hand side of (3.40) is proportional to

$$p_\mu \epsilon(p_0)\delta(p^2) \tag{3.41}$$

Thus corresponding to the existence of χ, there exists a momentum space delta function singularity in the theory.

4. THE ROLE OF THE GOLDSTONE BOSON

An important observation and a comment are in the sequel. The Fourier transform of (3.34) contains

$$\frac{2}{\pi}\lim_{\tau\to 0+}\left\{\theta(p_0)\delta'(p^2-\tau)+\frac{\pi}{2}\ln\tau\delta^4(p)\right\}+(2\ln 2-2\gamma)\delta^4(p) \tag{4.1}$$

wher γ is the Euler constant. δ' is not a measure, it arises as a consequence of the non-unitarity of translations. It becomes unitary in the negative metric space. Note that the primordial Lagrangian (3.1) was invariant under the transformation (3.2b). Ultimately, we have obtained a $\delta(p^2)$ singularity in the Fourier transform of (3.40). χ has been assumed to be a localized operator such that $\langle\Omega|\delta\chi|\delta B|\Omega\rangle\neq 0$ provided it is invariant under translations and the support of the Fourier transform is obtained in the forward light cone. But since the unitarity requirement condition $B^{(+)}|\text{phys}\rangle=0$ must always be valid, (4.1) is meaningful (Kibble, 1967) only in the limit of $\tau^+=0$, i.e., when the Fourier transform of (3.34) is proportional to $\delta(p^2)$. In order to elucidate the role of the Goldstone boson further we can use the relation (2.8) which has actually set up a connection between the Goldstone boson and the Ward identity. In doing so the Goldstone boson is constrained to the characteristic that it is present but that always in the limit of zero momentum. Let us recall that χ and ψ are all fields constructed out of ϕ and ϕ^*, and that we have familiar relations

$$[Q,\phi]=g_0\phi(x) \tag{4.2a}$$

and

$$[Q,\phi^*]=-g_0\phi(x) \tag{4.2b}$$

In the relation (2.8) let us choose $A=T(\phi(0)\phi^*(x))$. Then since ϕ's are

Hermitian fields, using (4.2) we can write

$$g_0\{\langle\Omega|T\phi(0)\phi^*(x)|\Omega\rangle+\langle\Omega|T\phi(0)\phi^*(x)|\Omega\rangle\}$$
$$=(2\pi)^{3/2}F^0\langle q|T\phi(0)\phi^*(x)|\Omega\rangle \tag{4.3}$$

It is a relation between the propagator Δ_F (left-hand side) and the scattering vertex (right-hand side) where

$$(2\pi)^{3/2}\int d^4x\, e^{-ipx}\langle\mathbf{q}|T\phi(0)\phi^*(x)|\Omega\rangle=\Delta_F(p-q)\Gamma(q,p)\Delta(p) \tag{4.4}$$

Therefore, the Fourier transform of (4.3) can be written in the form of Ward identity

$$g_0(\Delta_F+\Delta_F)=F^0\Delta_F(p)\Gamma(0,p)\Delta_F(p) \tag{4.5}$$

For on-the-mass-shell ϕ's, $p^2=\mu_0^2$ and Γ has the simple form

$$\Gamma(0,p)=g_0 \tag{4.6}$$

In view of (4.5) we have

$$F^0g_0=\mu_0=\mu/2^{1/2} \tag{4.7}$$

We have arrived at a striking result that the Goldstone boson in the theory connects F^μ as given by (2.7) to the Higgs meson mass and the bare coupling constant via (2.8).

5. CONCLUSION

The χ comes from a zero-mass one-particle intermediate state but is obtainable only at the limit of zero momentum. Now in view of equations (3.17) and (3.25) a question may be framed is that whether χ carries any superselection quantum number. Since we are actually interested in (4.7), following Haag and Kastler (1964) we can argue that the vacuum sector, containing only states of zero charge, does in fact contain all the information about the states carrying superselection quantum numbers and, observable themselves, cannot create particles carrying superselection quantum numbers. Let us consider the sequence of states in the vacuum sector $\mathcal{H}_\Omega$ representing a particle in the fixed open set O, together with an antiparticle further and further away. To an observer in O, the end of the sequence looks more and more like the one-particle state O. Similarly, states of the charge 2, 3,... should be constructed in $\mathcal{H}_\Omega$. Following Haag and Kastler (1964) we shall argue that, although the different coherent subspace H_q may carry mutually inequivalent representations π_q of the algebra G, all representations are physically equivalent in the following sense. Any real experiment will measure the expectation values $f_1, \ldots, f_n$, say, of a finite number of observables $A_1, \ldots, A_n$ with a finite error $<\delta$ and therefore establishes that the states could be any ν such that

$$|\nu(A_i)-f_i|<\delta, \qquad i=1,2,\ldots,n$$

If ν is a vector state in a representation π_q, then there will be a vector Ψ in the vacuum space $\mathcal{H}_\Omega$ such that

$$|\nu(A_i)-\langle\Psi|\pi_\Omega(A_i)|\Psi\rangle|<\delta \qquad \text{for } i=1,\ldots,n$$

It is not possible to distinguish the sets $\mathcal{H}_q$ and $\mathcal{H}_\Omega$ by making only a finite number observations.

In the theory we may put $M=0$, the theory survives and shows that a hand-inserted mass (m_0) is compatible with the existence of a Goldstone boson. Since the Goldstone boson invoked in the model does not increase the degree(s) of freedom of any of the fields present in the theory ($m_0 \neq 0$), there is no reason to demand a gauge-fixing condition of the form $\chi=0$ to do away with the Goldstone particle. Moreover, it is known to us that a massive vector field may be made to separate via a Stückelberg decomposition into a theory containing the transverse field and the accompanying longitudinal part. It is possible to recognize this longitudinal mode as a part of some scalar field (in the zero-mass limit this longitudinal mode decouples from the theory and may be treated as a Goldstone boson). Actually, introducing its partner (real) scalar field the theory may be shown to evolve into a gauge-invariant configuration with a charged matter field and one is then led to the question whether every massive vector field corresponds to some spontaneous symmetry breaking. We also note that if the Higgs meson mass $\mu=0$, the theory does not support spontaneous symmetry breaking. In fact, then, in (2.7), $\rho_1=\rho_2=0$. The question therefore raised is: should the χ be regarded as a function of the particle masses present in the theory? We propose to address it as *a class of Brans–Dicke scalars* of the scalar tensor theory of gravitation. A Brans–Dicke scalar ϕ is coupled to the trace T^μ_μ of the matter field through the relation $\Box\phi=8\pi\lambda T^\mu_\mu$, where λ is the coupling constant. In the present model this relation is invoked through the simultaneous validity of $\Box\chi=0$ and $F^0 g_0=\mu_0$, where the χ is in the limit of zero momentum.

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